

VECTOR ALGEBRA
CLASS XII

1. If $a = i + j + k$ and $b = j - k$, then find a vector c such that $a \times c = b$ and $a \cdot c = 3$
(Ans: $\frac{1}{3}(5i + 2j + 2k)$)
2. a, b, c are unit vectors, suppose $a \cdot b = a \cdot c = 0$ and angle between b and c is $\frac{\pi}{6}$. Prove that $a = \pm 2(b \times c)$.
3. Points L, M, N divide the sides BC, CA and AB of a ΔABC in the ratio $1:4, 3:2$ and $3:7$ respectively. Prove that $AL + BM + CN$ is a vector parallel to CK where K divides AB in the ratio $1:3$.
4. Find μ so that the vectors $2i - j + k, i + 2j - 3k$ and $3i + \mu j + 5k$ are coplanar.
(Ans: -4)
5. For any two vectors a and b , prove that $(a \times b)^2 = a^2 b^2 - (a \cdot b)^2$.
6. If $\alpha = 3i - j$ and $\beta = 2i + j - 3k$ then express in the form $\beta = \beta_1 + \beta_2$ where β_1 is parallel to α and β_2 is perpendicular to α .
(Ans: $(\frac{3}{2}i - \frac{1}{2}j) + (\frac{1}{2}i + \frac{3}{2}j - 3k)$).
7. If a, b, c be the vectors such that $a + b + c = 0, |a| = 3, |b| = 4$ and $|c| = 5$. Find $a \cdot b + b \cdot c + c \cdot a$.
(Ans: -25)
8. Using vectors show that the points $A(-1, 4, -3), B(3, 2, -5), C(-3, 8, -5)$ and $D(-3, 2, 1)$ are coplanar.
9. Show that the angle between any two diagonals of a cube is $\cos^{-1}(\frac{1}{3})$.
10. If a and b are two unit vectors and θ is the angle between them, then show
That $\sin \frac{\theta}{2} = \frac{1}{2}|a - b|$.
11. Find a vector whose magnitude is 3 and which is perpendicular to the following two vectors.
 $a = 3i + j - 4k$ and $b = 6i + 5j - 2k$. (Ans: $2i - 2j + k$)
12. If $a = 4i + 5j - k, b = i - 4j + 5k$ and $c = 3i + j - k$, find a vector d , which is perpendicular to both a and b and $d \cdot c = 21$.
(Ans: $7(i - j - k)$).